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VIBRATIONAL CHARACTERISTICS OF SOME THIN-WALLED CYLINDRICAL AND CONICAL FRUSTUM SHELLS*

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SUMMARY

Results of an experimental investigation of the vibrational characteristics of thin-walled, circular, cylindrical, and conical frustum shells with free-free and fixed-free boundary conditions are presented and compared with results obtained by using various analytical methods. The shells studied included two right circular cylinders differing in both length and diameter and a series of four conical frustums differing in cone angle but having a constant ratio of length to smaller-radius.

Experimental results show that free-free conical frustums have higher shell modes wherein a greater number of circumferential waves occur at the larger diameter than at the smaller diameter. The frustums having the greater conicity exhibit the greater differences in number of waves.

The data obtained indicate good agreement between experimental and theoretical natural frequencies and mode shapes for cylinders with both free-free and fixed-free boundary conditions and for conical frustums with fixed-free ends.

INTRODUCTION

The widespread application of cylindrical and conical frustum shells in space flight structures emphasizes the need for means to predict the dynamic response of such shells to applied forces. Much attention has been devoted to the dynamic behavior of thin-walled cylinders (e.g., refs. 1, 2, and 3), but considerably less information on the response of conical frustum shells to dynamic loading has been published (ref. 4).

Frequency equations for conical frustums have been derived only for certain boundary conditions. Shulman (ref. 5) assumed various displacement and stress functions in the evaluation of several approaches for obtaining natural

*Some of the information presented herein was previously included in the paper "Vibrational Characteristics of Thin-Wall Conical Frustum Shells" (AIAA Preprint 64-78), presented by the authors at the AIAA Aerospace Meeting, New York, N.Y., January 20-22, 1964.

frequencies and mode shapes of simply supported conical frustums. The analysis of Grigolyuk (ref. 6) for the simply supported conical shell is based on an energy approach using trigonometric mode shapes which reduce, in the limit, to those of the cylinder. Hermann and Mirsky (ref. 7) employed a similar approach for conical frustums of limited conicity (i.e., semivertex angles up to 15°). Federhofer (ref. 8) obtained approximations for the natural frequencies of cones and frustums of wide vertex angle in an analysis utilizing certain power expressions of the radius for the assumed longitudinal mode shapes. Natural frequencies of fixed-free frustums are predictable by the analysis of Saunders and Wisniewski (ref. 9), in which the radial displacement mode shape is represented by the simplest polynomial expression which will satisfy requirements of the fixed-end boundary condition. The theory of Platus (ref. 10) satisfies the fixed-end boundary condition more accurately than the method of Saunders and Wisniewski and also predicts considerably lower natural frequencies. McGrattan and North (ref. 11) have developed an analysis which predicts a difference in number of circumferential waves along the longitudinal axis for radially stiffened frustums fixed at the major diameter and subjected to a longitudinal driving force.

The present paper presents the results of an investigation of the vibrational characteristics of cylinders and conical frustums with free-free and fixed-free boundary conditions. Experimental data obtained for a series of cylindrical and conical frustum shells with these boundary conditions are compared with calculated results obtained from the Rayleigh type analyses of references 3, 10, and 12. Some of the calculations are based on an equivalent-cylinder approach suggested by Grigolyuk in reference 6, in which the frequencies of simply supported conical frustums of semivertex angles up to 15° are shown to be satisfactorily approximated by replacing the conical frustum with a cylindrical shell of radius equal to the average of the radii of the frustum and of length equal to the height of the frustum.

SYMBOLS

E Young's modulus of elasticity, psi

f natural frequency, cps

g gravitational constant, in./sec^2

l length of cylinder or frustum (see table 1), in.

m longitudinal mode number; number of circumferential nodes for free-free models, one more than the number of circumferential nodes between the ends of the model for fixed-free models

n circumferential mode number or number of circumferential waves

r_o cylinder radius; smaller radius of conical frustum (see table 1), in.

r_1 larger radius of a conical frustum (see table 1), in.

t shell thickness, in.

α semivertex angle of conical frustum, deg

Δ dimensionless frequency parameter defined by equation (4)

$\delta = r_1/r_o$

$\lambda = l/r_o$

μ Poisson's ratio

ρ_m mass density of shell material, γ/g, $\frac{\text{lb-sec}^2}{\text{in.}^4}$

ρ_w weight density of shell material, $\frac{\text{lb}}{\text{in.}^3}$

Subscripts:

e extensional

i inextensional

APPARATUS AND TESTS

Description of Models

Six thin-walled shells were tested: four right circular conical frustums and two right circular cylinders. Pertinent dimensions are given in table 1. Each shell was constructed of three identical sections of 0.007-inch-thick stainless steel with 5/32-inch overlapped, spotwelded, longitudinal seams. The larger radius of each frustum was held constant, while dimensions of the smaller radius and the length were adjusted to maintain a ratio λ of 3.0. The frustums consequently had semivertex angles which varied from 3.2° to 24.0°. The ratio of length to radius of cylinder 1 was 3.0, whereas the corresponding ratio for cylinder 2 was 1.8.

Description of Model Support System

Free-free supports.- As a means of simulating free-free boundary conditions, the models were suspended by six strings spaced evenly about the circumference of the shell as shown in figure 1.

Fixed-free supports.- As a means of simulating fixed-free support, frustums 1, 2, and 3 were rigidly fixed at the smaller diameter by the clamping action of the two plates shown in figure 2(a). The smaller base of the conical frustum was placed in the 3/4-inch-deep tapered cutout of the larger plate and then the small plate was bolted to the larger plate. A clamping action was thus obtained between the frustum and the two plates. A series of clamps spaced evenly around the circumference of the larger plate attached the assembly to a massive backstop. In order to satisfy the requirements of zero slope and deflection at the fixed end, it was necessary to imbed frustum 4 and both cylinders in a metal alloy as shown in figure 2(b).

Instrumentation

Each model was excited with either an air shaker (described in ref. 13) or with one or two electromagnetic shakers (fig. 1) capable of a maximum output force of 1.3 pounds each. The instrumentation used in conjunction with the electromagnetic shakers is shown schematically in figure 3. The electromagnetic shakers were driven by the amplified output from an audio oscillator. An electronic counter measured the frequency of the oscillator output. A separate potentiometer was used to adjust the output to each shaker individually. One of two transducers, either a crystal accelerometer or a velocity probe, was used to determine the response of the structure to induced vibration. The amplified transducer-output signal was applied to the vertical plates of a cathode-ray oscilloscope, while the oscillator-output signal was applied to the horizontal plates so that a Lissajous pattern resulted on the scope.

Measurement of Natural Frequencies and Mode Shapes

In tests using the air shaker natural frequencies were obtained by varying the excitation frequency until maximum specimen response resulted as observed visually or by fingertip touch. Nodal patterns were also determined in this manner.

Natural frequencies during tests with the electromagnetic shakers were excited by varying the oscillator frequency until observation of shell-displacement amplitudes indicated maximum specimen response. Distinct Lissajous patterns were then obtained by slight adjustment of the excitation frequency. Nodal patterns were determined by moving the transducer over the shell surface and observing phase relations between the oscillator and transducer signals.

ANALYTICAL DETERMINATION OF NATURAL FREQUENCIES AND MODE SHAPES

This section presents the equations used to predict the natural frequencies of the free-free and fixed-free shells considered in this investigation.

Free-Free Boundary Conditions

Cylinders.- Natural frequencies of the free-free cylindrical shells were calculated by using the analyses of references 3 and 12. The Timoshenko ring equation (ref. 12) for flexural vibrations in the plane of the ring, with twist neglected, is applicable when $m = 0$ and is given by

$$f = \frac{1}{2\pi}\left[\frac{Eg}{\rho_w}\,\frac{t^2}{12r_o^4}\,\frac{n^2(1 - n^2)^2}{1 + n^2}\right]^{1/2} \tag{1}$$

Analytical results were also calculated by using the analysis of reference 3, for $m = 0$, 1, and 2, in which the radial, circumferential, and longitudinal motions of the shell are each approximated by products of trigonometric functions in the circumferential direction and elementary beam functions in the longitudinal direction. A cubic frequency equation is obtained in which the contributions of the longitudinal motions of the shell to the mode shapes could be varied in accordance with a given set of boundary conditions.

Conical frustums.- An adequate analysis has not been developed for prediction of the natural frequencies and mode shapes obtained experimentally for the free-free conical frustums under investigation.

Fixed-Free Boundary Conditions

Cylinders.- Natural frequencies of the fixed-free cylindrical shells were calculated by using the analysis of reference 10 in addition to that of reference 3 which was previously applied to the free-free cylinders.

The analysis of reference 10 is a Rayleigh Ritz type in which frequency equations for inextensional (bending or flexural) vibrations and for extensional (membrane) vibrations are derived and linearly combined. In the extensional frequency equation the longitudinal component of the mode shape is approximated by a power series involving as many as six terms each in radial, circumferential, and longitudinal directions, so that an 18th-degree frequency equation results. The extensional frequency parameter Δ_e is related to the extensional frequency f_e as follows:

$$\Delta_e = \frac{(1 - \mu^2)\rho_m\lambda^2 l^2(2\pi f_e)^2}{E} \tag{2}$$

The inextensional frequency equation of reference 11 is based on a two-term approximation to the longitudinal component of the mode shape and is given by

$$f_i = \frac{tn(n^2 - 1)}{4\pi r_o^2}\left[\frac{E}{3\rho_m(1 - \mu^2)}\right]^{1/2}\left[\frac{n^2 + 6(1 - \mu)\left(\frac{r_o}{l}\right)^2}{3\left(\frac{r_o}{l}\right)^2 + n^2(n^2 + 1)}\right]^{1/2} \tag{3}$$

Replacing the subscript e in equation (2) with the subscript i yields Δ_i, the inextensional frequency parameter.

Thus, theoretical values for the parameter $(\Delta)^{1/2}$ were obtained by calculating Δ_e with a digital computer program (ref. 10), by calculating Δ_i from equations (2) and (3), and by forming $(\Delta)^{1/2}$ from the relation

$$(\Delta)^{1/2} = (\Delta_i + \Delta_e)^{1/2} \tag{4}$$

Conical frustums.- The analytical procedures of reference 10 were also employed to calculate frequency parameters for the fixed-free conical frustums. The extensional frequency parameter Δ_e was found by use of a computer program analogous to that applied to the cylindrical shells. Inextensional frequency parameters were calculated from equation (2) after first computing inextensional natural frequencies from the equation

$$f_i = \frac{tn(n^2 - 1)}{4\pi r_o^2 \cos\alpha}\left[\frac{E}{3\rho_m(1 - \mu^2)}\right]^{1/2}\left[\frac{N}{D}\right]^{1/2} \tag{5}$$

where

$$N = \left(1 - \frac{\sin^2\alpha}{n^2}\right)^2 \log_e \delta - \frac{2}{\delta}(\delta - 1)\left(1 - \frac{\sin^2\alpha}{n^2}\right) + \frac{(\delta^2 - 1)}{2\delta^2} + \frac{(1 - \mu)(\delta^2 - 1)\sin^2\alpha}{n^2\delta^2} \tag{6}$$

and

$$\begin{aligned} D = {} & \frac{1}{2}(\delta^2 - 1)\left(1 + \frac{n^2}{\cos^2\alpha} + \frac{\tan^2\alpha}{n^2} - 2\tan^2\alpha\right) \\ & - \frac{2}{3}(\delta^3 - 1)\left(1 - \frac{\sin^2\alpha}{n^2}\right)\left(1 + \frac{n^2}{\cos^2\alpha} - \tan^2\alpha\right) \\ & + \frac{1}{4}(\delta^4 - 1)\left(1 - \frac{\sin^2\alpha}{n^2}\right)^2\left(1 + \frac{n^2}{\cos^2\alpha}\right) \end{aligned} \tag{7}$$

The frequency parameter $(\Delta)^{1/2}$ was then obtained by substituting values of Δ_i and Δ_e into equation (4).

Equivalent cylinder calculations were made with the analytical procedure of reference 3 for the fixed-free frustums. The equivalent cylinder for a given frustum was defined by setting the cylinder radius equal to the average of the radii of the frustum, as recommended in reference 6.

RESULTS AND DISCUSSION

The following section consists of two parts. Experimental results of the investigation are presented in the first part, while the second part is devoted to a discussion of the comparison between calculated and measured natural frequencies.

The indices m and n identify both the experimental and the analytical mode shapes. (See fig. 4.) The circumferential mode number n refers to the number of circumferential waves (or 1/2 the number of longitudinal node lines) for both free-free and fixed-free boundary conditions. The longitudinal index m is equal to the number of circumferential nodes for free-free boundary conditions, whereas for fixed-free conditions it is one more than the number of circumferential nodes between the ends, the clamped end not counting as a node.

Natural frequencies, both experimental and theoretical, are presented in terms of the dimensionless frequency parameter defined by equation (4), in which either the calculated or the experimental natural frequencies, whichever is appropriate, is substituted to obtain the corresponding frequency parameters.

Presentation of Data

Natural frequencies obtained for a given model by using the air shaker are compared in table 2 with those obtained by exciting the shell with an electromagnetic shaker. These data indicate that the natural frequencies obtained by using the electromagnetic shaker were usually higher than those excited by using the air shaker. The maximum increase was 6.1 percent. When the electromagnetic shaker was used, natural frequencies below 7 cps were not recorded because of the inaccuracy of instrumentation at these low frequencies.

Effect of certain structural asymmetries.- An additional model having the same dimensions as cylinder 2 but with four seams was tested to determine what effect an additional seam would have on the natural frequencies. The results (table 3) show that the four-seam model tended to have slightly higher natural frequencies for a given mode than did the three-seam model; the maximum increase was 4.8 percent.

The effect of any out-of-roundness of a model on its natural frequencies and mode shapes was also investigated experimentally. A free-free frustum was pulled out-of-round to form an elliptical cross section. A comparison of natural frequencies and mode shapes between this configuration and the original model with the same free-free suspension indicated that out-of-roundness had no appreciable effect upon either natural frequencies or mode shapes.

Free-free shells.- Variations of the frequency parameter with the number of circumferential waves are shown in figures 5(a) and 5(b) for the free-free cylindrical shells for $m = 0$, 1, and 2. The trends are similar to those found in reference 14.

Experimental results for the free-free conical frustums appear in figure 6. These results show that at higher natural frequencies there are a greater number of circumferential waves at the larger diameter than at the smaller diameter. The difference in the number of waves increased as the natural frequencies of a given shell increased. The difference in the number of waves also increased with conicity. The difference ranged from 1 to 5 waves for the frequency range covered in the investigation. (See fig. 6(d).) These trends were found to be independent of the type shaker used to induce vibration.

Figure 7 shows nodal patterns typical of those obtained for the free-free frustums. These particular patterns were obtained for frustum 3 with 6 and 8 circumferential waves at the smaller and larger diameters, respectively. While in general two electromagnetic shakers tended to excite free-free modes more distinctly, the nodal patterns obtained tended to be asymmetrical (fig. 7(a)). Patterns obtained by using a single electromagnetic shaker, while not as distinct, tended to be symmetrical (fig. 7(b)).

Fixed-free shells.- The variation of measured and calculated frequency parameters with the number of circumferential waves is shown in figure 8 for the fixed-free cylindrical shells and in figure 9 for the fixed-free conical frustums. The trends for the cylindrical shells are similar to those found in reference 15, and the trends for the conical frustums are similar to those found in reference 16.

Comparison of Measured and Calculated Natural Frequencies

Frequency parameters calculated for the free-free cylindrical shells utilizing either the Timoshenko ring frequency equation ($m = 0$) or the analysis of reference 3 ($m = 0$, 1, 2) are in good agreement with experimental results, as may be seen in figures 5(a) and 5(b). As was found in reference 3, Timoshenko's equation gives frequencies that are in closer agreement with measured values than does the analysis of reference 3. Although the minimum measured frequency for cylinder 1 could not be obtained, calculations indicate the existence of a minimum frequency at $n = 11$.

The analytical procedures of references 3 and 10 yield results which are in good agreement with experimental data obtained for the fixed-free cylindrical shells with longitudinal mode number $m = 1$. The results obtained with analytical procedures of reference 3 for $m = 2$ are in good agreement with experimental results for cylinder 2, whereas the agreement is only fair for cylinder 1.

Good agreement was also obtained between experimental and analytical results for fixed-free frustums in the vicinity of minimum frequency for all cone angles tested. As the conicity increases, however, the agreement becomes poorer at higher circumferential mode numbers.

Results of equivalent-cylinder calculations for the fixed-free frustums are compared with experimental results in figure 10. This approximation is adequate only for the smallest cone angle, 3.2°. Thus, whereas the equivalent cylinder may be adequate for simply supported conical frustums for cone angles up to 15°, as noted in reference 6, it was not satisfactory for the clamped-free frustums of the present study that had cone angles greater than 3.2°.

CONCLUDING REMARKS

The results of the study show that the natural frequencies predicted by Rayleigh type vibration analyses are in good agreement with experimental results for cylindrical shells with either free-free or fixed-free boundary conditions and also for conical frustums with fixed-free ends. An adequate theory for prediction of the natural frequencies and mode shapes for free-free conical frustums has not yet been developed.

An equivalent cylinder analogy was found to be applicable to fixed-free frustums having semivertex angles of 3.2° or less. As this angle increases, however, the agreement of this analogy with experimental results becomes increasingly poor.

It was determined experimentally that the mode shapes associated with the higher natural frequencies of free-free conical frustums exhibit a greater number of circumferential waves at the larger diameter than at the smaller diameter. This difference in number of waves increased as the order of the mode or natural frequency increased for a given shell.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., September 15, 1964.

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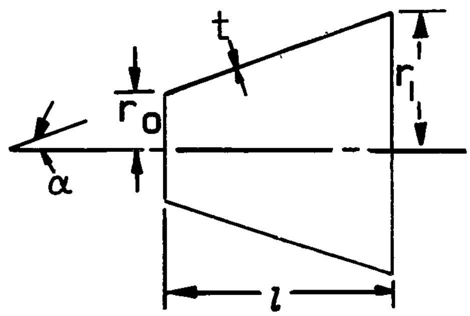
TABLE 1.- STRUCTURAL PARAMETERS OF MODELS



Structure	α, deg	t, in.	l, in.	r_o, in.	r_1, in.
Cylinder 1	0	0.007	42	14	14
Cylinder 2	0	.007	18	10	10
Frustum 1	3.2	.007	36	12	14
Frustum 2	7.4	.007	30	10	14
Frustum 3	14.0	.007	24	8	14
Frustum 4	24.0	.007	18	6	14

TABLE 2.- EFFECT OF TYPE OF SHAKER ON NATURAL FREQUENCIES OF A CIRCULAR CYLINDRICAL SHELL WITH FREE-FREE ENDS

[Cylinder 1; r_o = 14 in.; l = 42 in.]

m	n	Natural frequency, cps	
		Air shaker	Electromagnetic shaker
0	2	0.9	----
0	3	2.5	----
0	4	4.8	----
0	5	7.6	7.6
0	6	10.9	10.8
0	7	15.5	16.4
0	8	20.3	20.9
0	9	26.1	26.4
0	10	32.3	32.6
1	3	3.9	----
1	4	6.3	----
1	5	9.1	9.5
1	6	12.1	12.5
1	7	16.4	17.4
1	8	20.9	22.0
1	9	26.8	27.4
1	10	32.8	33.1

TABLE 3.- EFFECT OF NUMBER OF SEAMS ON NATURAL FREQUENCIES OF A CIRCULAR CYLINDRICAL SHELL WITH FREE-FREE ENDS

[Cylinder 2; r_o = 10 in.; l = 18 in.]

m	n	Natural frequency, cps	
		Three seams	Four seams
0	2	1.8	1.8
0	3	4.8	4.8
0	4	9.1	9.3
0	5	14.9	15.3
0	6	21.7	22.3
0	7	30.4	31.1
0	8	40.1	40.9
0	9	51.0	52.1
0	10	63.2	64.6
1	3	6.6	6.7
1	4	10.9	11.1
1	5	17.4	17.3
1	6	23.1	24.2
1	7	33.0	33.5
1	8	43.1	43.9
1	9	52.7	54.3
1	10	65.5	67.4

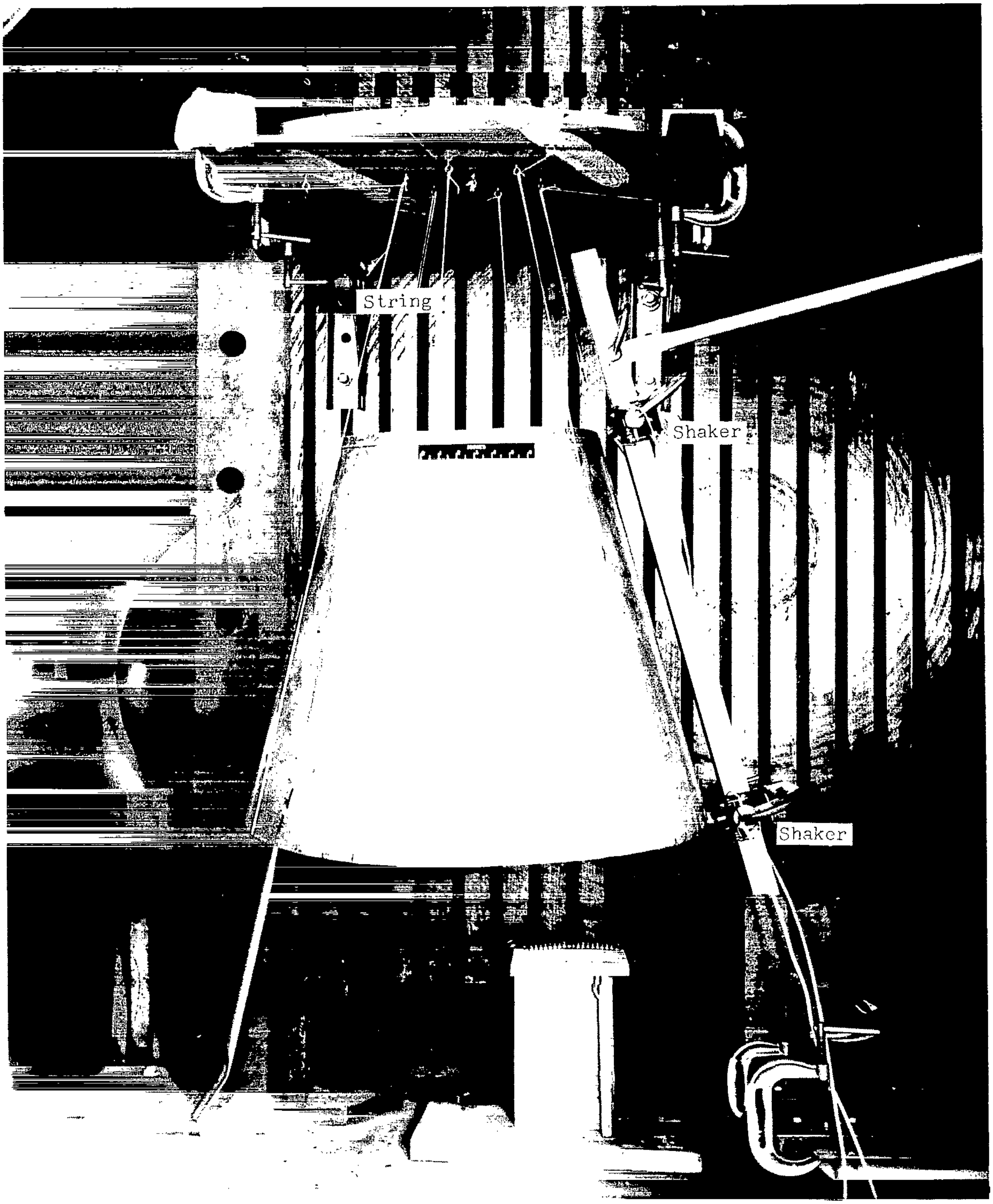
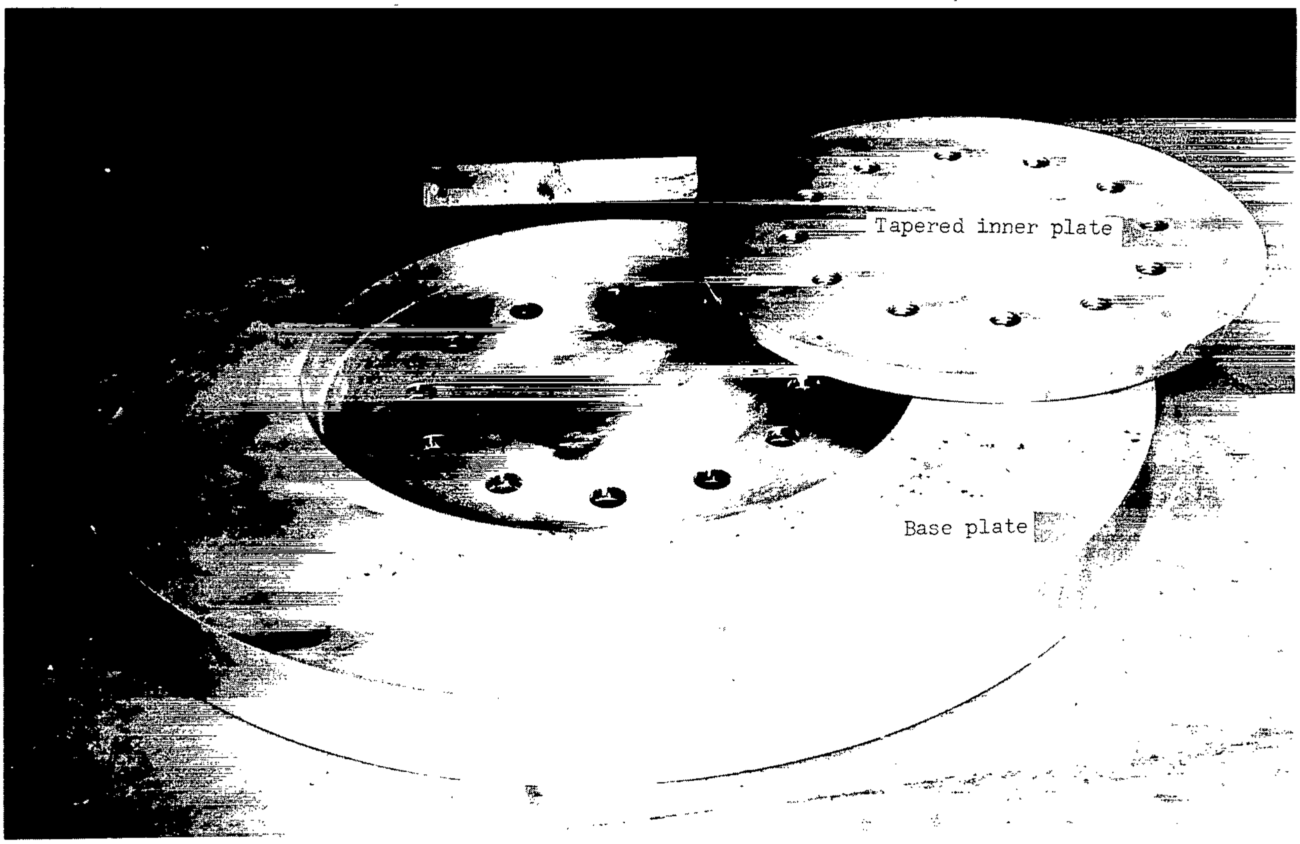
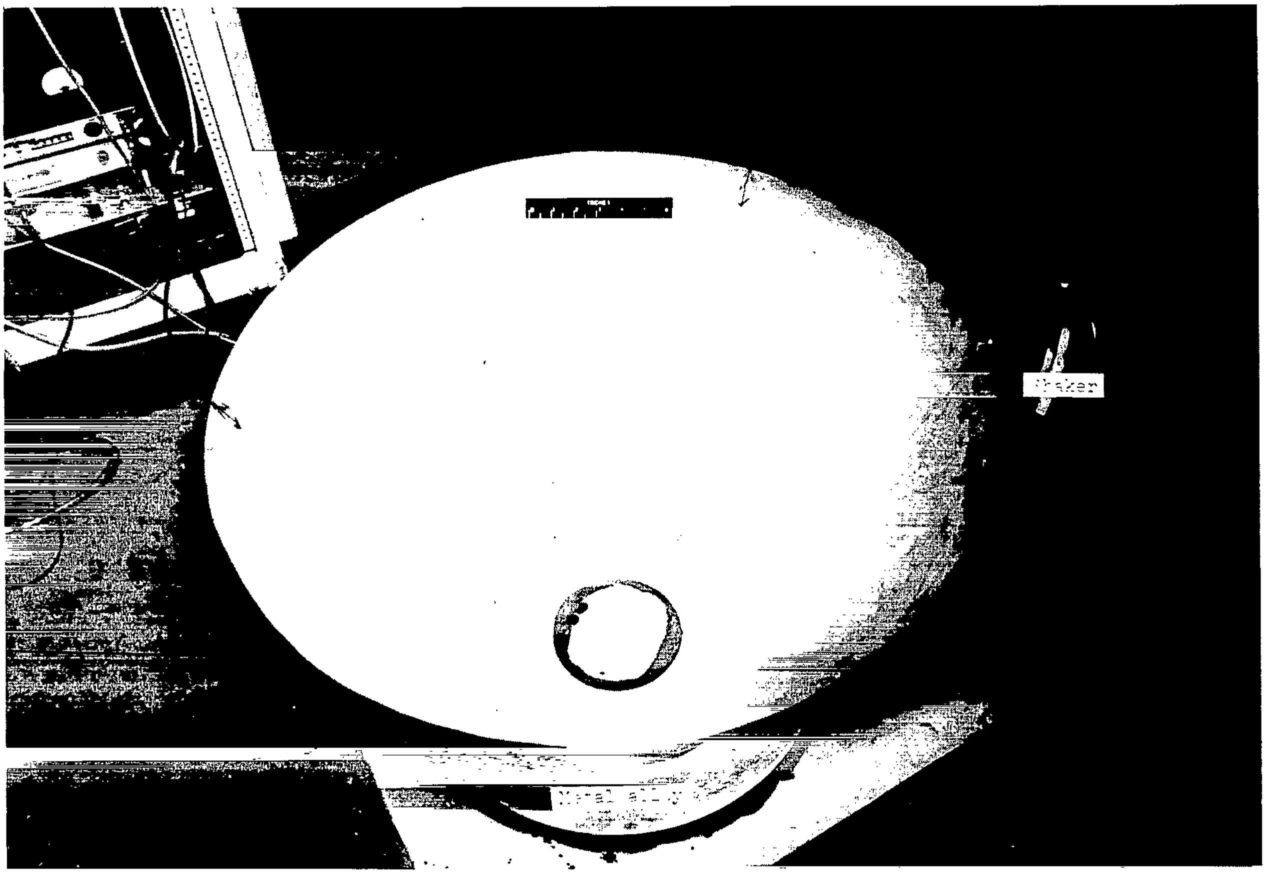


Figure 1.- Suspension system for free-free boundary conditions. L-63-8911.1



(a) Clamped with two plates. L-63-5633.1

Figure 2.- Support system for fixed-free boundary conditions.



(b) Imbedded in metal alloy. L-63-5635.1

Figure 2.- Concluded.

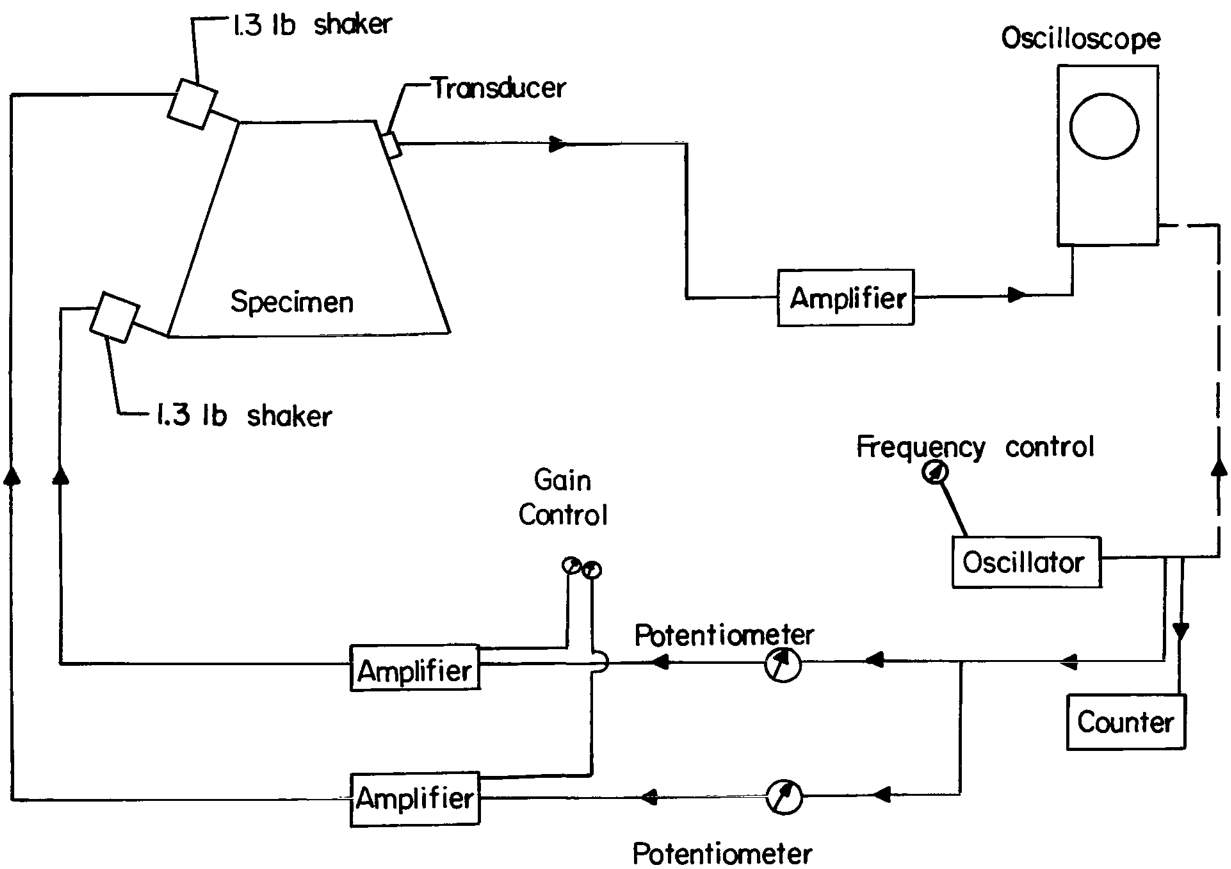
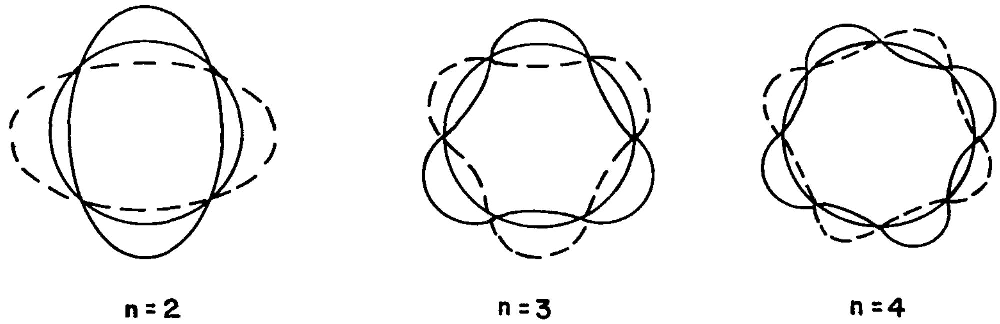
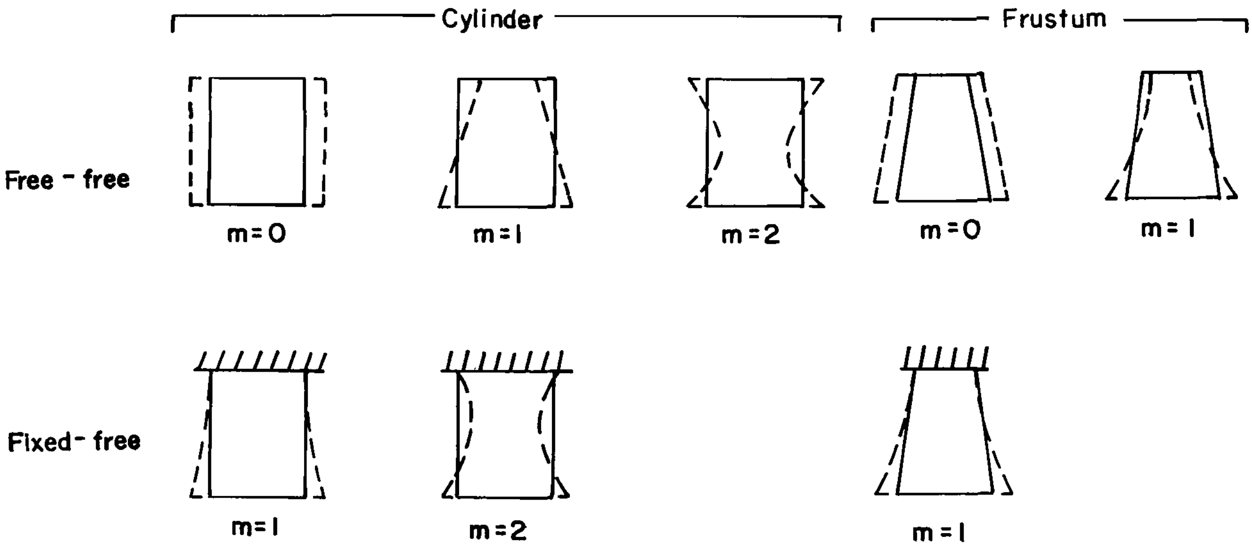


Figure 3.- Instrumentation block diagram.

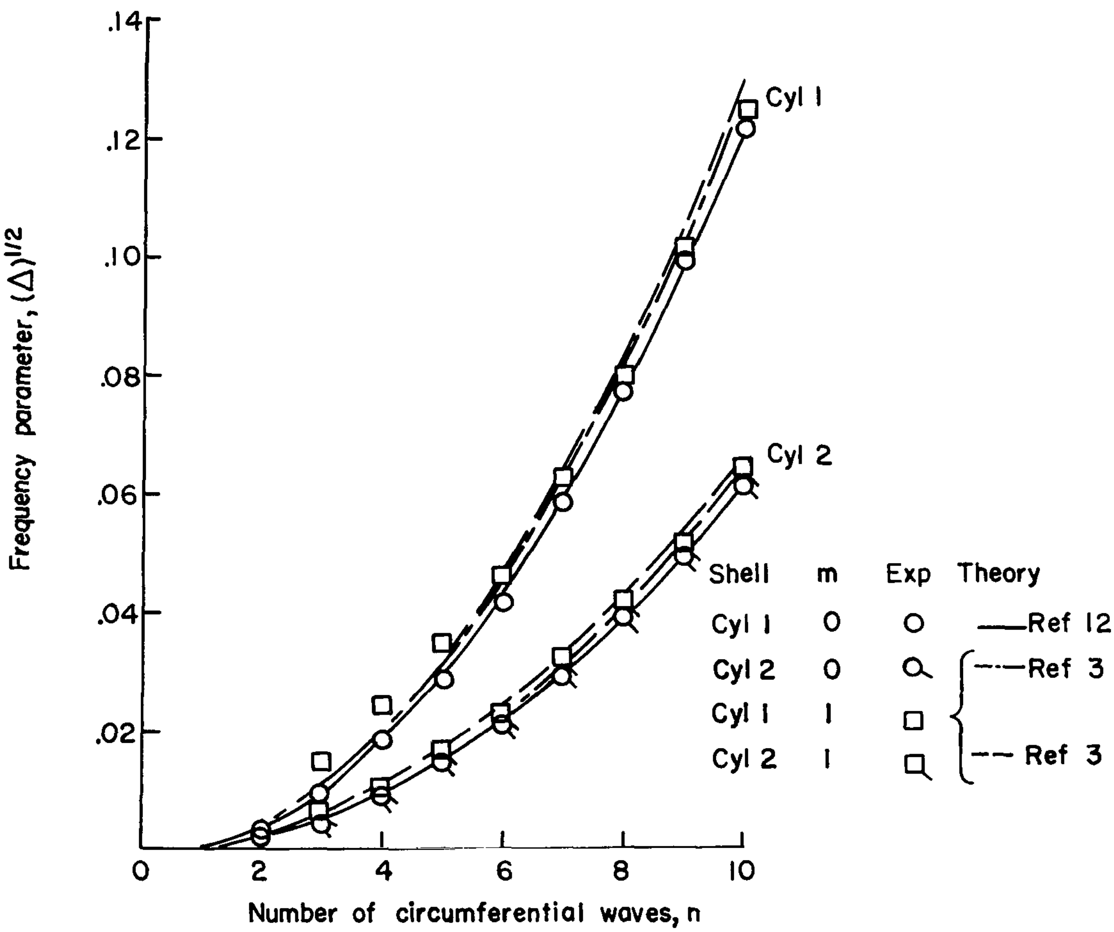


(a) Cross sections depicting circumferential waveforms.



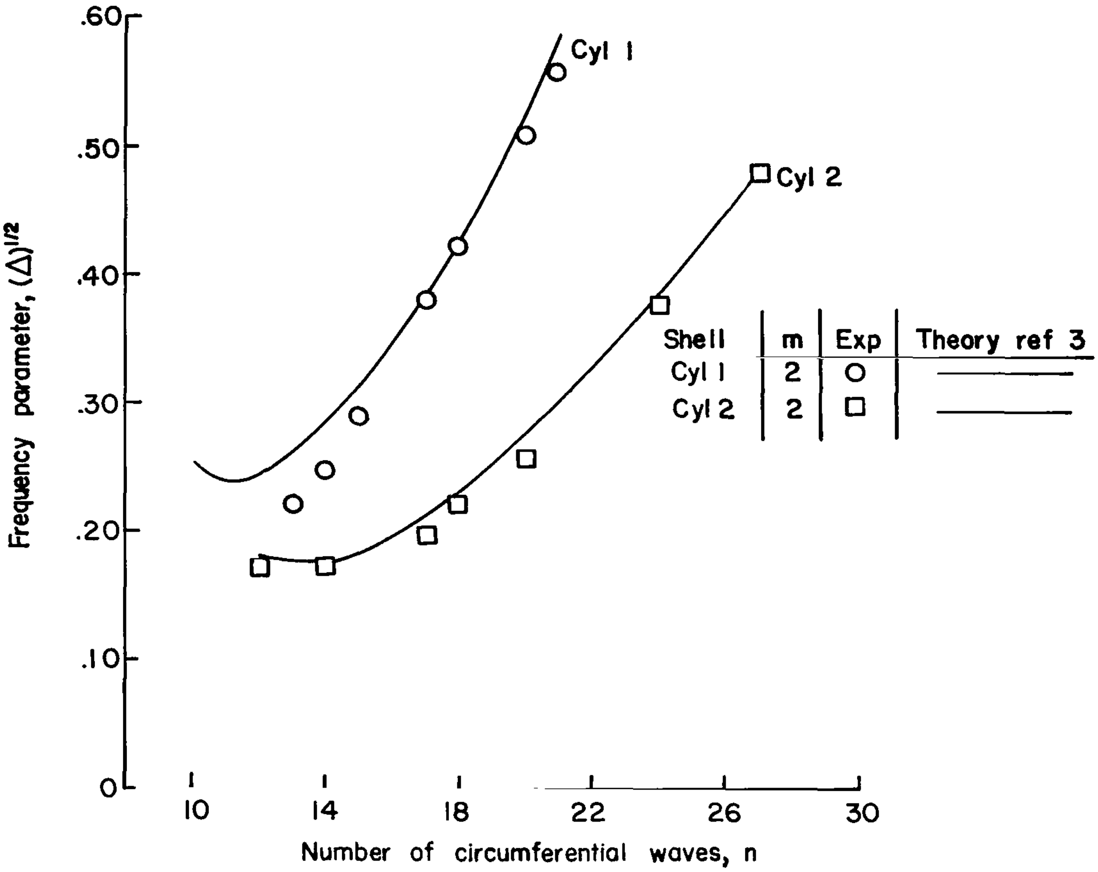
(b) Cross sections depicting longitudinal waveforms.

Figure 4.- Deflection patterns of radial vibrations of thin-walled cylindrical and conical frustum shells.



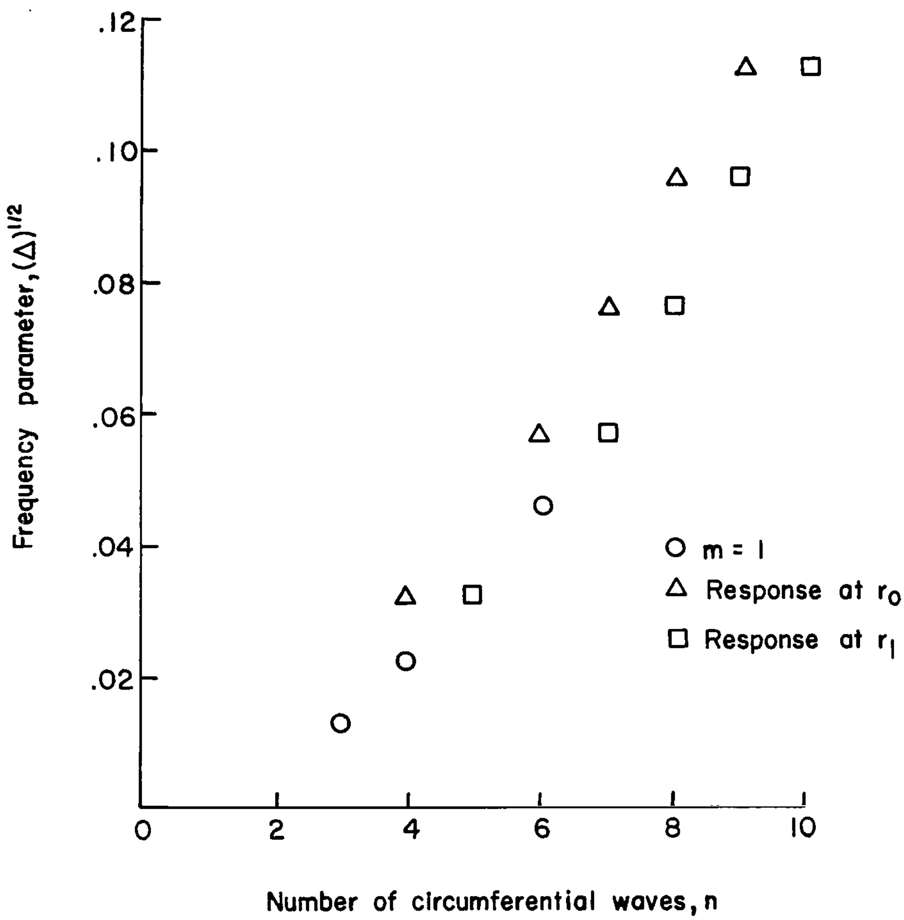
(a) m = 0; 1.

Figure 5.- Frequency characteristics of circular cylindrical shells with free-free end conditions.



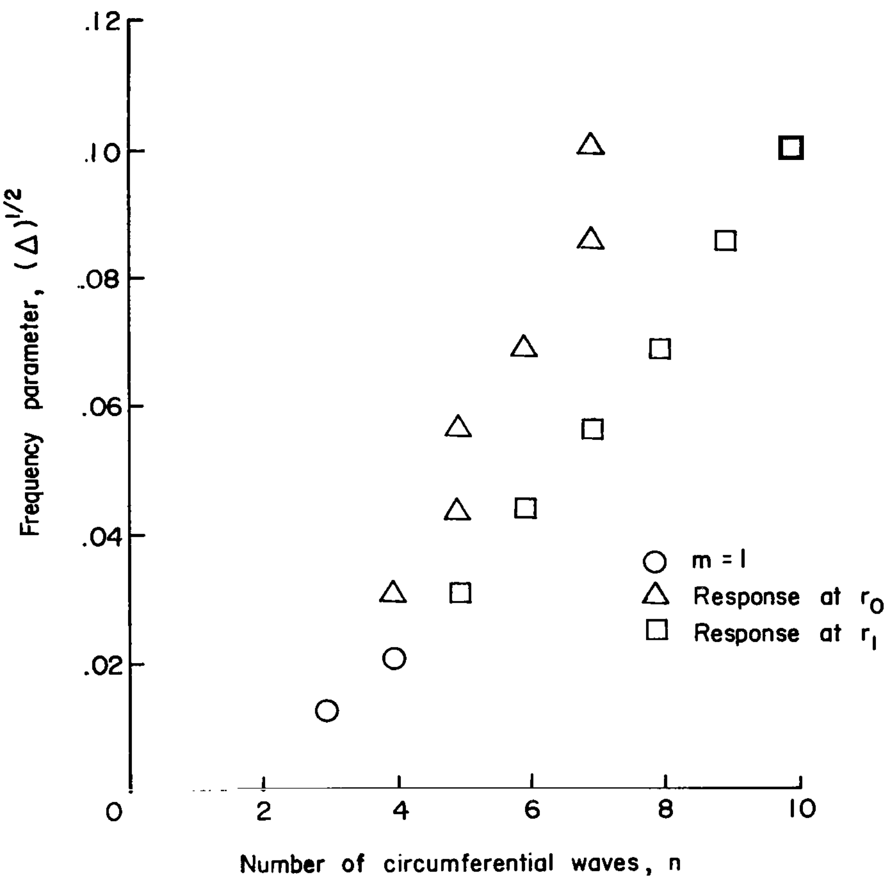
(b) m = 2.

Figure 5.- Concluded.



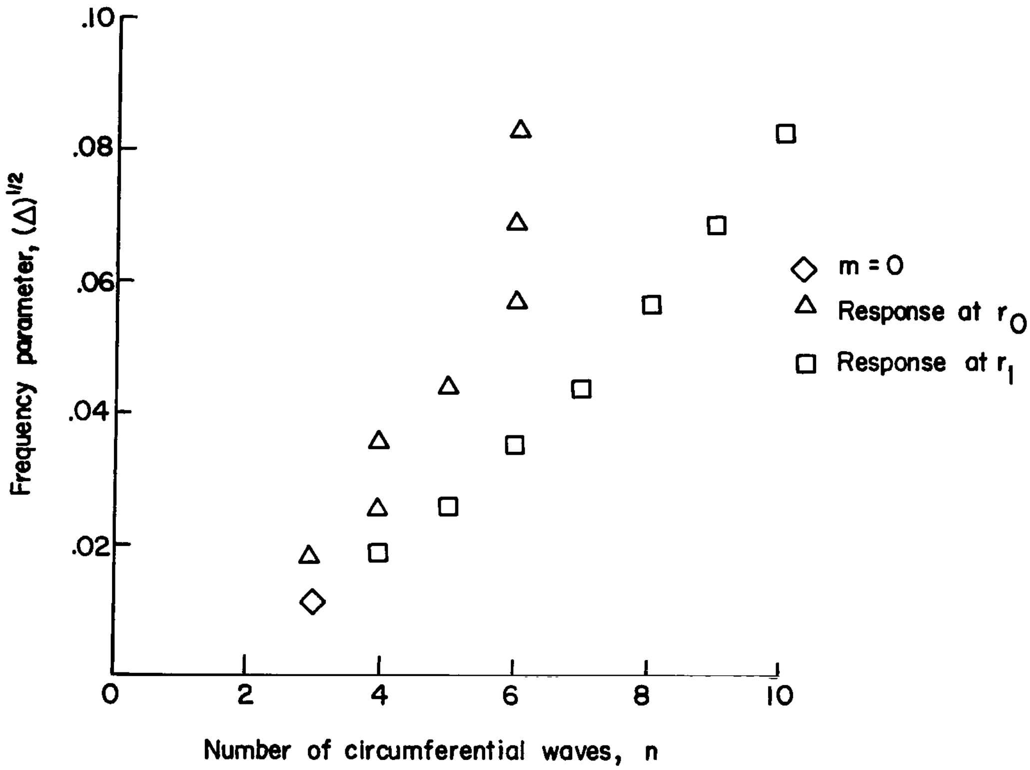
(a) $\alpha = 3.2^{\circ}$.

Figure 6.- Experimental natural frequency characteristics of a circular conical frustum with free-free end conditions.



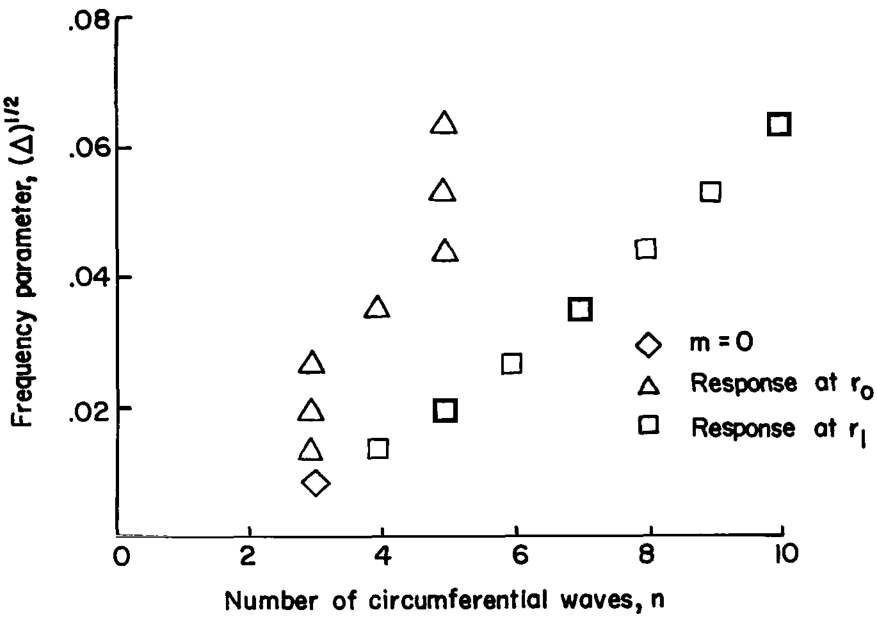
(b) $\alpha = 7.4^{\circ}$.

Figure 6.- Continued.



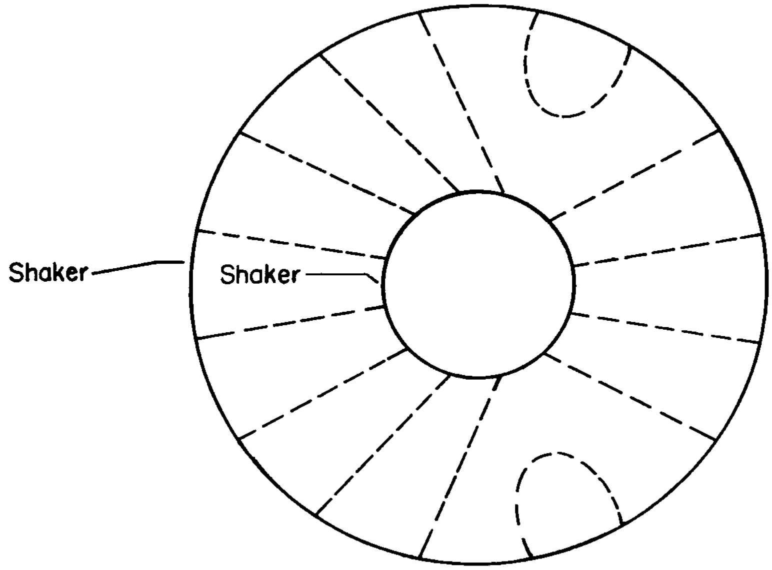
(c) $\alpha = 14.0^{\circ}$.

Figure 6.- Continued.

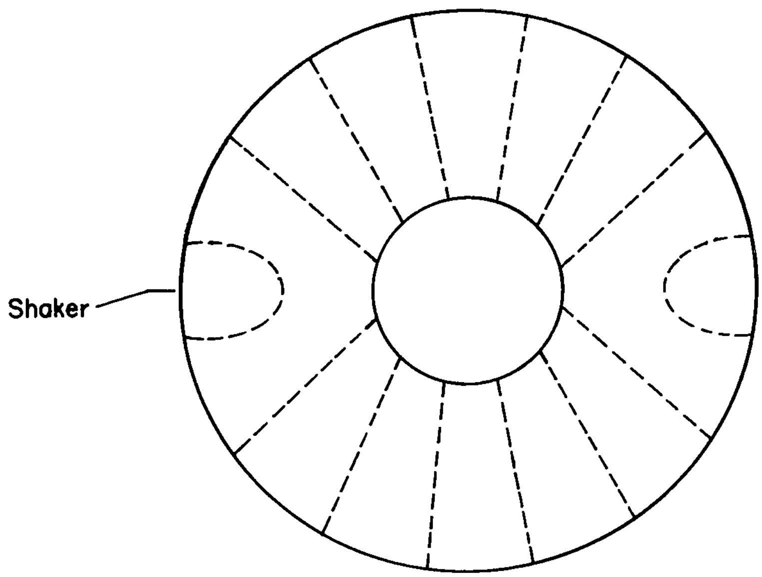


(d) $\alpha = 24.0^{\circ}$.

Figure 6.- Concluded.



(a) Asymmetrical nodal pattern.



(b) Symmetrical nodal pattern.

Figure 7.- Sketch of typical nodal patterns for free-free frustums as viewed along longitudinal axis for frustum 3.

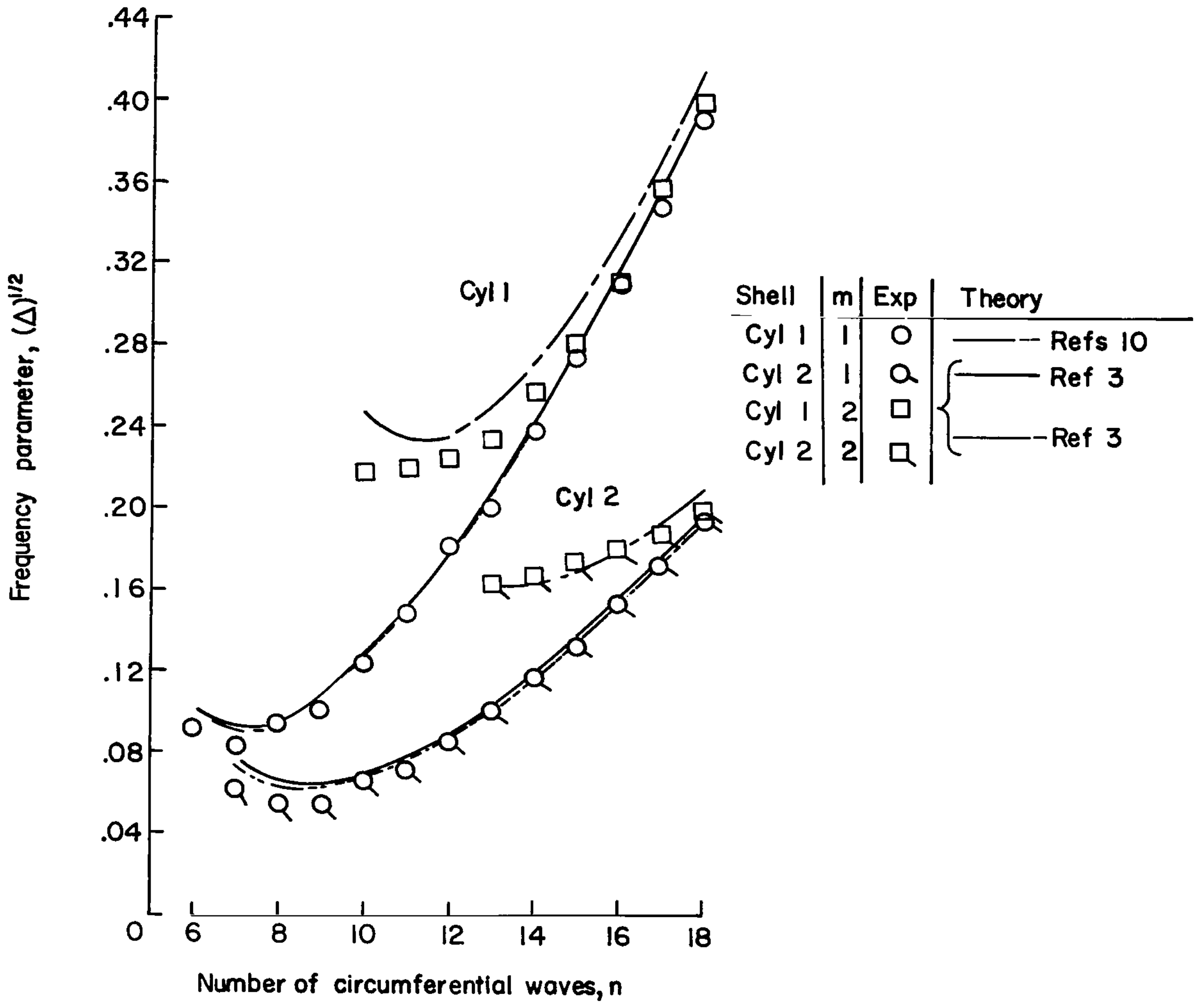


Figure 8.- Frequency characteristics of circular cylindrical shells with fixed-free end conditions. m = 1; 2.

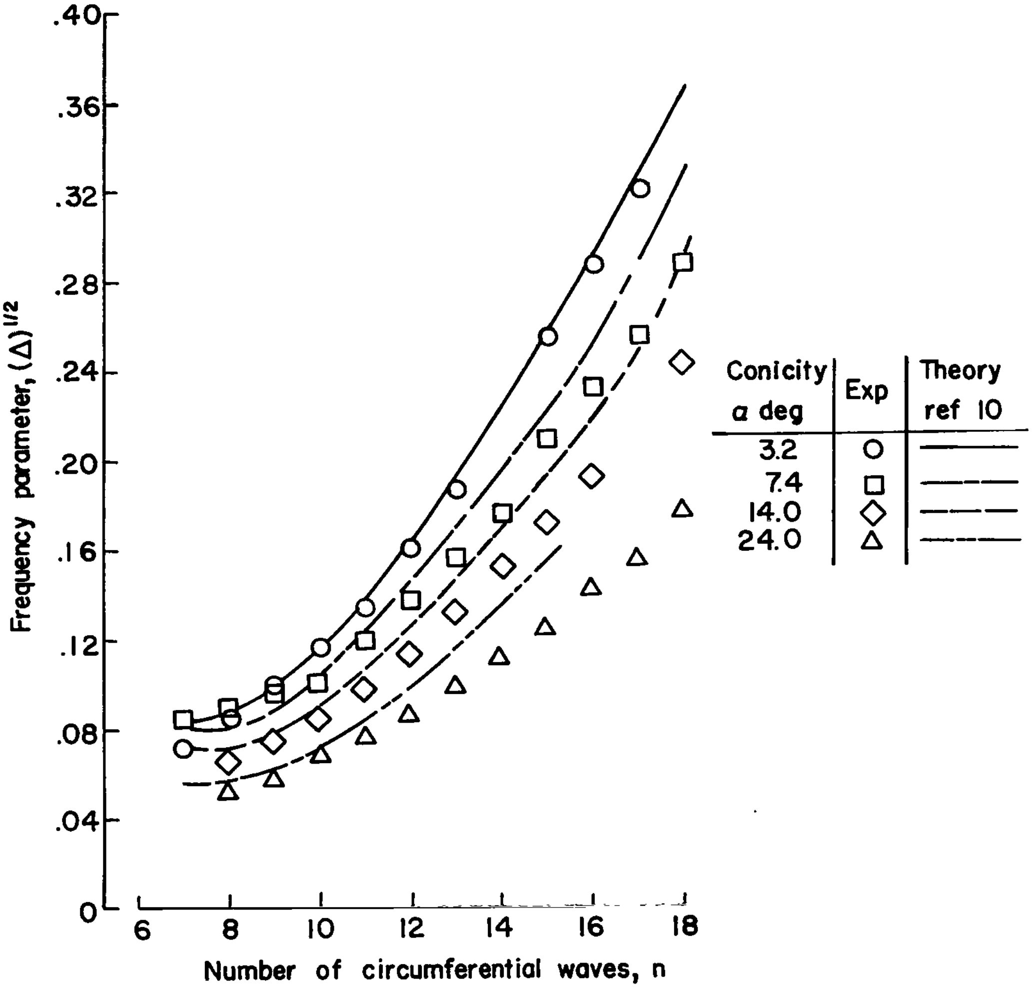


Figure 9.- Frequency characteristics of conical frustum shells with fixed-free end conditions. m = 1.

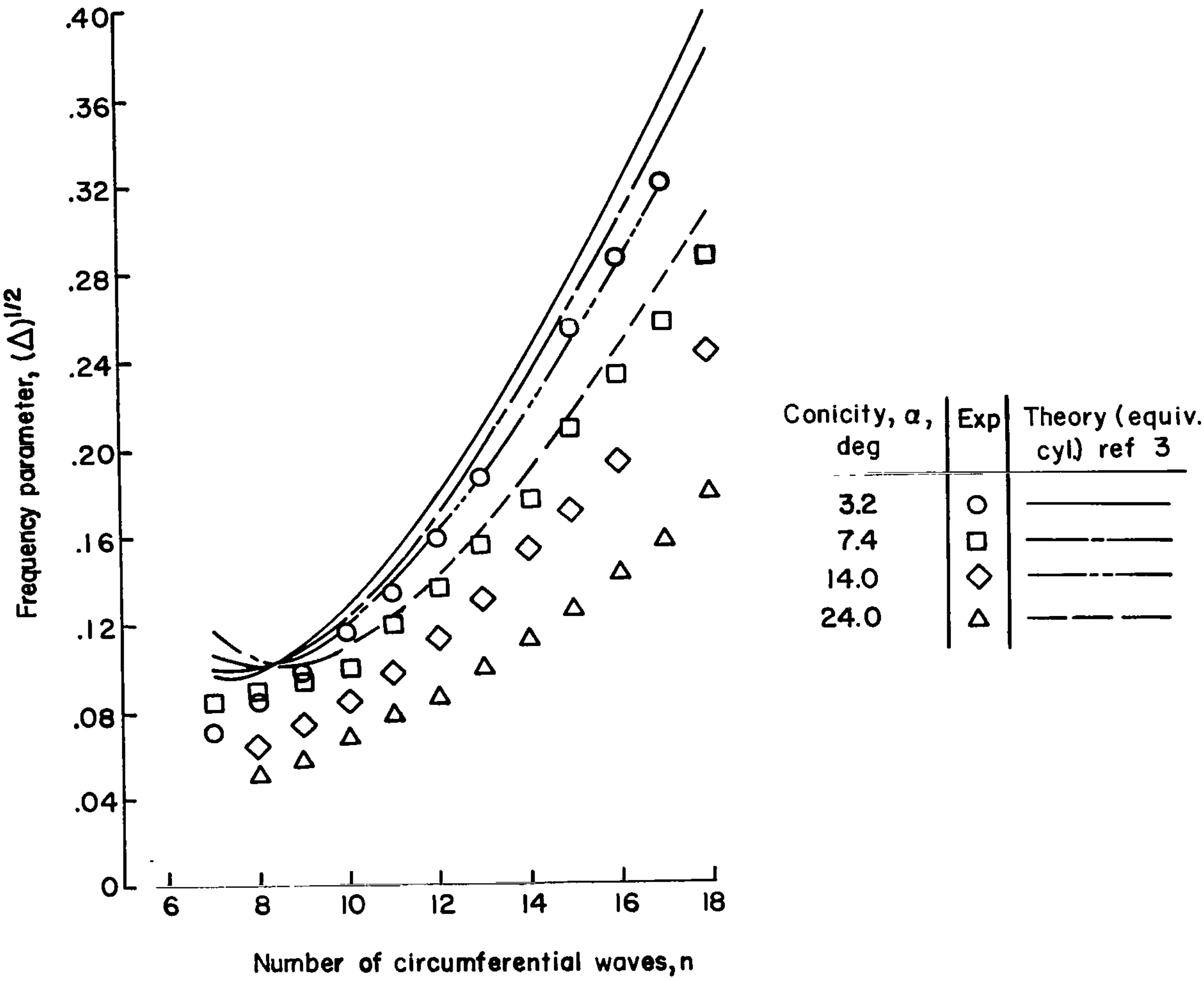


Figure 10.- Comparison of experimental frequency parameter for conical frustums with calculated values for an equivalent cylinder $\left(\text{Radius of equivalent cylinder} = \frac{r_1 + r_o}{2}\right)$. $m = 1$.

2/22/85 JZ

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